Adaptive Behavior

Linking Micro- to Macro-level Behavior in the Aggressor–Defender–Stalker Game

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In many multiagent systems, small changes in individual-level rules may lead to very large changes at the group-level. This phenomenon is striking in the "aggressor-defender game," a simple participative game in which each participant randomly selects two others from the group (*A* and *B*). In the aggressor game, everyone tries to position themselves so that *A* is always between themselves and *B*. In the defender game everyone tries to position themselves between *A* and *B*. Despite these exceedingly simple rules and the seemingly small difference between them, the two games exhibit very different dynamics. The aggressor game produces a highly dynamic group that rapidly expands over time whereas the defender game quickly collapses to a tight knot. I analyze these games and provide some insight as to how these two group level behaviors arise, thereby linking the micro- and macro-levels. I also introduce and analyze a new, related and simpler game, the stalker game, in which each participant selects and pursues a single participant, and which also produces a collapsing group. It is suggested that such a geometrical analysis may be applicable for other multiagent systems such as insect societies and collective robotics.

Keywords swarming · self-organization · participative game · aggressor · defender · stalker

1 Introduction

Possibly the greatest challenge in the analysis and design of complex systems is to understand the link between the micro-level and macro-level (Anderson, in press). That is, how do the individual level rules, and the interactions among those individuals and the environment, give rise to the far more complex, emergent properties and system-level behavior? And, conversely, how does the group level dynamics feedback to the lower level and affect those individuals?

Complex systems are characterized by a network of interactions and feedbacks (often positive) with non-linear group-level dynamics (e.g., Casti, 1994; Lewin, 1995; Coveney & Highfield, 1996). Thus, even a complete knowledge of the proximate rules and mechanisms employed at the individual level may provide little insight as to the group-level dynamics that will emerge. Coupled with this is the confounding effect that small changes in individual level rules may, through the amplifying effects of the feedbacks, translate to very different group-level behavior (e.g., Wilson, 1975; Ünsal, 1993).

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The best way for students to appreciate this important property of complex systems is to experience it. A simple, clear and fun demonstration is provided by the "aggressor–defender" game, an extremely basic participative game dating at least to the Fratelli Theater Group at the 1999 *Embracing Complexity* conference, and recently promoted and developed by Bonabeau and colleagues (Bonabeau, 2002a,b; Bonabeau & Meyer, 2001; Bonabeau et al., 2003a,b; Funes, Orme, & Bonabeau, 2003). In the aggressor subgame, every participant selects two others at random from the group, say *A* and *B* (and maintains these choices throughout the game), and attempts to position themselves so that *A* is always between themselves and *B*—imagine this as a defender *A* protecting you from an aggressor *B*. In the defender subgame, everyone tries to position themselves between *A* and *B*—imagine this as you defending *A* against *B*.

(By "game" we mean "a competitive and often good humored pastime played to particular rules" (ikjeld.com/info/glossary/glossaryG.html) or "a physical or mental competition in which the participants, called players, seek to achieve some objective within a given set of rules" (www.xs4all.nl/~mgsch/gaming/theory_glossary.htm) rather than, for example, those of game theory in which a set of discrete actions are associated with a payoff. In short, we mean the colloquial term rather than the academic term.)

Despite these simple rules, and the seemingly small differences between them, the two games exhibit very different dynamics. The aggressor game produces a highly dynamic group that expands over time whereas the defender game quickly collapses to a tight knot—see http://www.icosystem.com/game.htm for an online demonstration (see also Bonabeau, Funes, & Orme, 2003a; Anderson, 2003; see Palmer et al., 2003a,b, for similar human swarming games). Taking an intuitive, geometrical approach, I provide some significant insight as to how these two group level patterns occur, thus linking micro- to macro-level behavior. I also introduce and analyze a related, simpler game which I term the "stalker game" in which each participant selects and pursues just a single participant and which also produces a collapsing group, as well as other more surprising behavior.

The motivation behind this work is twofold: first, to understand the micro- to macro-level mappings in the aggressor–defender–stalker games specifically. While the individual level rules are very simple, the fact that they implemented as a *social network* makes it nontrivial to understand how they lead to the global observed patterns. In other words, these games are a very useful example of a complex system such that the collective patterns are not obvious a priori yet a certain degree of analysis is possible. Second, and more generally, to expound this particular geometrical approach as it may be applicable, relevant, and useful for analyzing other multiagent systems. That is, there are likely many situations in which each "agent" directly interacts solely with one or two members of a larger group and the interactions are relatively simple. Such systems may include collective robotics, swarms of unmanned air vehicles (UAVs) or underwater vehicles (UUVs), insect societies, and even some human activity. For instance, a form of stalker game with multiple interactions certainly exists at parties and conferences when one attempts to catch up with a particular friend or colleague while someone else attempts to catch up with you. While there are currently no concrete, rigorous, and objective criteria by which one can decide whether such an approach is useful for a given system, this study is a useful first step in illustrating the sort of analysis and insights that may be gained by such a geometricallyoriented approach.

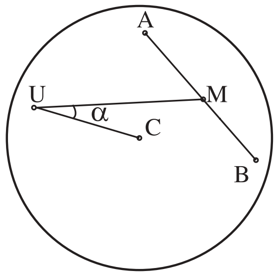

Figure 1 The main components of the model: the focal individual (*U*), the complete set of participants (the disc with center *C*), and *U*'s two selections, *A* and *B*. In the defender game, *U* heads for *M* which is the midpoint of the line *AB*. Angle α represents *U*'s deviation from the center when heading to *M*.

2 Defender Game

Rather than deal with a finite number of people in an ill-defined space, we assume a well organized, infinite number of participants, now points, uniformly distributed on a disk of unit radius and center *C* (Figure 1). Consider random points *A*, *B* and *U* on the disk and assume that the game rule is that *U* attempts to head to the midpoint *M* of a line joining *A* and *B*. When

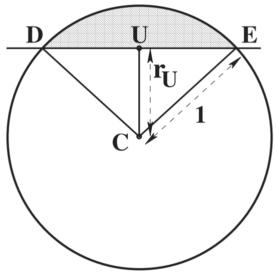

Figure 2 The hatched area represents position of *M* in which *U* is moving away from the center *C*. The non-shaded area of the circle is denoted $T(r_U)$.

$\angle MUC$, denoted α, is less than $\frac{\pi}{2}$ then *U* is moving closer to the center (at least initially). If $\alpha \geq \frac{\pi}{2}$, then *U* is heading away from the center. Showing that the expected value of α, $E(\alpha) \leq \frac{\pi}{2}$; partially demonstrates that individuals tend towards the center.

The real goal here is to calculate the joint probability density function (hereafter, pdf) of *M*, thereby mapping the initial distribution of participants, that is, uniformly distributed across a disk, to the first iteration of the game. (The mapping from where participants are to where they wish to be essentially implies instantaneous jumping.) Such a mapping will reveal whether the game is expected to produce an initial expansion or contraction. Unfortunately, however, writing down an integral covering all possible positions of *A*, *B* and *U* does not easily yield a closed-form solution for *M*'s joint pdf. Thus, I develop a more geometrical approach.

The symmetry of the situation implies that the distribution must be rotationally symmetric. Therefore, we need only consider a distance, say $r_U \in [0, 1]$, from the center along a typical radius rather across a disk. For a particular r_U, we take the line perpendicular to the radius at this point (that is, a line perpendicular to line CU). This line intersects the perimeter of the disk at positions *D* and *E* (Figure 2). The area beyond this line (that is, between r_U and 1) is the area in which $\alpha > \frac{\pi}{2}$ and the complementary area is where $\alpha \leq \frac{\pi}{2}$, hereafter area $T(r_U)$. Importantly, for all positive r_U, area $T(r_U) \geq 0.5$ thereby indicating that movement towards the center is favored. Further, because of the rotational invariance, this is true *whatever* the true underlying distribution of *M*. A special case of interest is if *M* were uniform on a disk, which would occur if the game rule were "choose one individual from the group at random and pursue them." Thus, I predict that this game, which I term the "stalker game," will also produce a collapsing group (see Section 4).

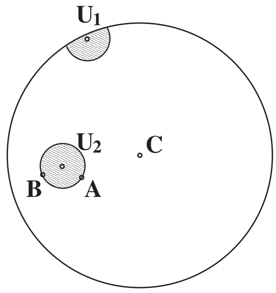

Figure 3 Myopic defenders. With low sight and hence a small complete disk around focal individuals, as in U_2, there will be no bias. When the individuals are close to the perimeter, as in U_1, then there will be a bias towards the center *C*.

2.1 Myopic Defenders

In the online simulation of the aggressor–defender game, one can vary the "sight" of an individual, that is, the participant's range of vision thereby limiting the locality from which they may choose *A* and *B*. With low sight, individuals are myopic, only able to see and choose from close neighbors. In the simulation, one observes a slow partial collapse of the group, which repeatedly fragments and fuses in several groups. At the individual level, however, movement is far more rapid and local. Following the above intuitive logic, one can develop some insight as to the group dynamics.

We can consider selection of *A* and *B* occurring from a small disk around random *U* (individual U_2 in Figure 3). Except where these "areas of sight" overlap the perimeter of the disk, there is no bias of *M* around *U*. For *U*'s at the perimeter however (individual U_1 in Figure 3), there will be a bias driving them somewhat closer to the center *C*. Thus, individuals at the perimeter but not internal individuals, cause the collapse of the group, albeit at a lower rate than for the long-range vision defender game. These peripheral individuals are expected to have a high turnover, similar to the swarm front of army ants (e.g., Franks, Gomez, Goss, & Deneubourg, 1991) in which the ants at the swarm move at a slower rate than the main trail.

While the "sight" or sensing range clearly has a large influence upon group dynamics, it is not, unfortunately, easily analyzed. Thus, for the remainder of the study, we assume infinite sensing range: That is, individuals may choose among all participants, even if they are located on the other side of the group with many individuals potentially blocking direct line of sight.

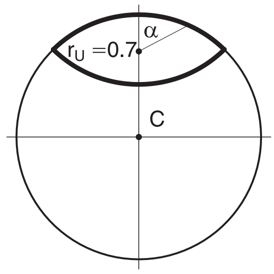

Figure 4 Given point $M = (r_U, 0)$, here $r_U = 0.7$, the thick solid line bounds the region for which each point *A* has a complement *B* such that *M* is line *AB*'s midpoint.

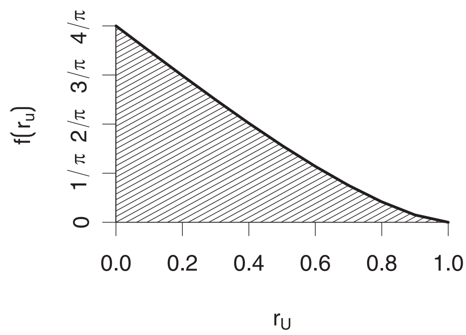

Figure 5 Normalized radial pdf (Equation 3) for the defender game.

2.2 Full Joint pdf of *M*

As *M*'s joint pdf is rotationally invariant we can calculate an unscaled "radial pdf" along a typical radius—this provides a relationship between the relative probability of *U* moving to that distance in the next iteration of the game. Then, this "pdf" is rotated about center *C* to produce a solid of revolution whose volume is used to normalize the whole joint pdf. First, we consider our focal *M* at $r_U = 0$. *Any* point *A* on the disk will have some complement *B* that has its midpoint as *M*. Thus, one can consider that there are π different, valid sets of $\{A, B\}$ with midpoint *M*. At the boundary, $r_U = 1$, there are no valid solutions as *A* must coincide with *B*. For intermediate r_U, we have a more complex situation as we lose some symmetry. For a given position on *M*, $0 < r_U < 1$, the region of possible positions of *A* for which we can find a *B* such that their midpoint is *M* is a pseudo-ellipse shown in Figure 4—one can verify this by imagining a line centered at *M* and rotating it through angle $\alpha \in [0, 2\pi]$.

Here, the "relative number of solutions," denoted $S(r_U)$, is twice the shaded area in Figure 2:

$$\begin{aligned} S(r_U) &= 2(\pi - T(r_U)) \\ &= 2(\cos^{-1}(r_U) - r_U\sqrt{1 - r_U^2}), \end{aligned} \tag{1}$$

where $r_U \in [0, 1]$. Thus, we have a function that gives us the *relative* heights of the joint pdf from $r_U = 0$ to $r_U = 1$. Next, we integrate about the center to get a solid of revolution. Letting *V* represent this solid's volume, then

$$V = 4\pi\int_0^1 r_U(\cos^{-1}(r_U) - r_U\sqrt{1 - r_U^2})\mathrm{d}r_U = \frac{\pi^2}{4}. \tag{2}$$

Therefore, in polar coordinates, the joint pdf of the first iteration of the defender game is

$$f(r_U, \theta) = \frac{8}{\pi^2}(\cos^{-1}(r_U) - r_U\sqrt{1 - r_U^2}), \tag{3}$$

where $r_U \in [0, 1]$ and $\theta \in [0, 2\pi)$. This is plotted in Figure 5.

2.3 Rate of Collapse of the Group

Figure 6a shows the rate of collapse for a single defender game of 100 simulated participants. This collapse is illustrated by plotting the area of the minimum convex polygon (MCP: The smallest convex polygon that encompasses a set of locations in 2D space) of the participants for each of the first ten iterations of the game. In figure 6b, the areas of these MCPs is plotted against iteration number for 20 replicates. It is clear that the rate of collapse of this game is exponential. The best fit to these data was $6.51e^{-0.7x}$ ($R^2 = 0.998$).

2.4 Conclusion

The above calculations show that, distributionally, the defender games essentially maps an initial uniform on a disk, i.e., a cylinder (radius 1, height $1/\pi$), to a cone (radius 1, height $4/\pi$). Clearly, after the first iteration, our typical individual *U* is not only likely to be closer to the center anyway, but its selections, *A* and *B*, will also be closer to the center. Thus, given the game rule, the distribution will condense even more in each subsequent iteration. While this is not a formal proof, it certainly provides significant insight as to why the group collapses.

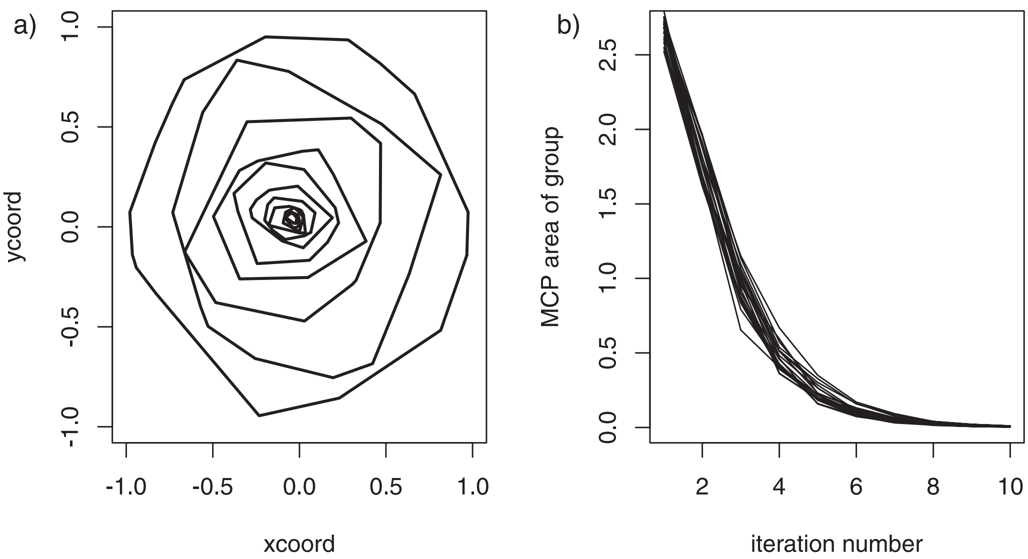

Figure 6 The rate of collapse for the defender game with $n = 100$ participants: (a) minimum convex polygons of the participants' locations for successive iterations of a single game, (b) area versus iteration number (20 replicates).

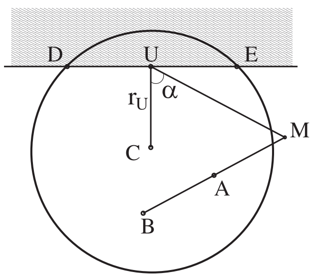

Figure 7 The aggressor model in which *A* is midway between *B* and *M*. *M* is no longer confined to the unit disk and therefore *U* is heading away from *C* whenever *M* is in the region above the line *DUE* (and hence $\alpha > \frac{\pi}{2}$).

3 Aggressor Game

In the aggressor game, *U* tries to position him/herself so that *A* is between *U* and *B*. Sticking with *M* representing *U*'s intended position, I assume that *A* is the midpoint between *M* and *B* (Figure 7; this assumption is relaxed in another paper: C. Anderson, in preparation), in other words, that the game rule is $\|M - A\| = \|A - B\|$.

In this game, the joint pdf of *M* is not confined to the initial disk of unit radius. In the extreme, with *A* and *B* lying opposite each other on the perimeter of the disk, and because of the condition that $\|M - A\| = \|A - B\|$, then *M* may occur in a disk up to *three* units radius from *C* (hence, after each iteration, or jump, the bounding area of *M*'s joint pdf expands *nine* times; this is, of course, an upper bound for the expansion). Thus, although the same properties of rotational invariance and $T(r_U) > 0.5$ (that is, bias towards the center) hold, what is very different from the earlier defender scenario is that individuals are likely to *overshoot* the original unit disk. So, even though there may some tendency towards the center and not outwards *per se*, the overshoot—that individuals may head through the center and head out many units radius from the center—means that the group likely will expand rapidly.

3.1 Full Joint pdf of M

To obtain an explicit form for the joint pdf, I adopt the same geometrical consideration as for the defender game: Obtain a radial pdf representing the relative probability of finding *M* at a given distance, here with $r_U \in [0, 3)$, and then normalize it with a solid of revolution. For $r_U \in [0, 1]$ *any* point *B* on the disk has a complement *A* for *any* point *M* on the disk. Therefore, the joint pdf is flat for $r_U \in [0, 1]$ with relative height π. For $r_U = 3$, there is a single solution: *A* on the unit disk's perimeter at the point that is closest to *M*, with *B* on the exact opposite of the unit disk's perimeter. As before, we can consider this as zero "solutions." Finally, we must consider the more tricky intermediate case $1 < r_U < 3$.

Consider along the positive horizontal axis. We are interested in pairs of points on the disk that satisfy two conditions: First, that a line passing through those

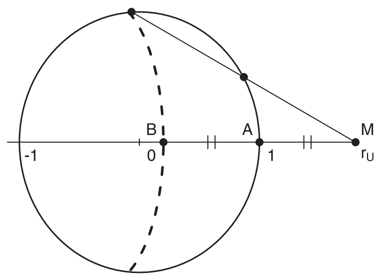

Figure 8 The unit disk and point $(r_U, 0)$. The dashed arc delineates valid solutions: to the right of this arc it is not possible to satisfy $\|B-A\| = \|A-M\|$ with A remaining on the disk. Therefore, the relative height of the pdf for a given r_U is the area within the unit disk but to the left of the arc.

points also passes through $M = (r_U, 0)$, and second that $\|B-A\| = \|A-M\|$ (where B is farther from M than A). For this horizontal line, the critical x-value is $2 - r_U$. That is, if $B = (2 - r_U, 0)$ then A must be on the margin of the disk, $A = (1, 0)$, so $\|B-A\| = \|A-M\|$ that which in this case equals $r_U - 1$ (see Figure 8). I call this the critical value because if B were any larger, A would have to be outside the disk. Thus, this critical value delineates the invalid set of solutions, to the right, and the valid ones to the left—whose area is the relative height of the pdf.

We sweep a line passing through M from gradient = 0 (running along the x-axis) to the gradient such that the points of intersection with the circle satisfy $\|B-A\| = \|A-M\|$ (beyond this gradient, $\|B-A\| < \|A-M\|$). As we sweep this line, we calculate the point on the line (and disk) which delineates the valid and invalid solutions. Our aim is to form an arc on the disk; to the right of this arc will be invalid solutions, a set of points B that do not have valid complements A (Figure 8). Next, we calculate the area on the disk to the left of this arc to give us the relative number of solutions for that given r_U. Finally, we repeat for all $r_U \in (1, 3)$.

The line $y = mx + c$ that passes through $(r_U, 0)$ intersects the unit circle, $y^2 + x^2 = 1^2$, at x-coordinates

$$x = \frac{r_U m^2 \pm \sqrt{1 + m^2(1 - r_U^2)}}{m^2 + 1}. \tag{4}$$

Hereafter, I refer to these two roots as x^+ and x^-.

The limit of the sweep is defined by m such that $\|r_U - x^+\| = \|x^+ - x^-\|$; that is, when

$$r_U - \frac{r_U m^2 + \sqrt{1 + m^2(1 - r_U^2)}}{m^2 + 1} = \frac{2\sqrt{1 + m^2(1 - r_U^2)}}{m^2 + 1},$$

which gives

$$m = -\frac{\sqrt{9 - r_U^2}}{3\sqrt{r_U^2 - 1}}. \tag{5}$$

At this value of m, $x^- = (3 - r_U^2)/2r_U$. This value sets the lower x-bound of the arc, and we have already determined the upper x-bound for the arc: $x = 2 - r_U$ when $m = 0$. To determine the whole arc, for a given r_U and m we find the x-coordinate such that $\|x^+ - x\| = \|x^+ - r_U\|$. This is simply $2x^+ - r_U$ which is

$$x = \frac{r_U(m^2 - 1) + 2\sqrt{1 + m^2(1 - r_U^2)}}{m^2 + 1}. \tag{6}$$

For that r_U, m, and x, the y-coordinate is $2(r_U - x^+)|m|$. Taking m as being negative,

$$y = \frac{-2m}{(m^2 + 1)}\left(r_U - \sqrt{1 + m^2(1 - r_U^2)}\right). \tag{7}$$

(This equation only provides positive values of y; however, the symmetry of the situation means that we can take $\pm y$ for each $|m|$ and r_U.) We now have x and y as two functions of r_U and m. Through substitution and rearrangement, we can obtain y as a function of x and r_U. This is,

$$y = -\frac{\sqrt{(r_U - x)^2}\sqrt{4 - r_U^2 - 2r_U x - x^2}}{2(1 - r_U x)} \left(r_U - \sqrt{\frac{(r_U^2 + r_U x - 2)^2}{(r_U - x)^2}}\right). \tag{8}$$

The relative number of solutions is thus the area of the disk from $x = -1$ to $x = (3 - r_U^2)/2r_U$ plus the area under the arc from $x = (3 - r_U^2)/2r_U$ to $x = 2r_U$. That is,

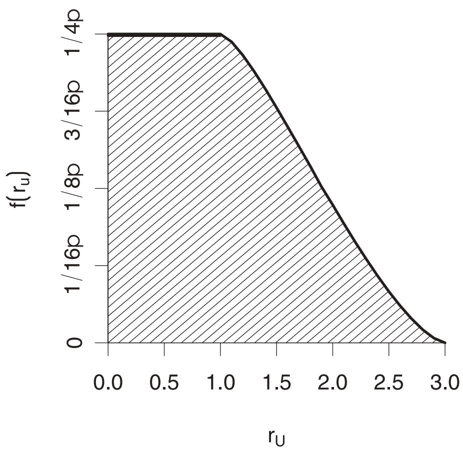

Figure 9 The normalized radial pdf of the aggressor game.

$$S(r_U) = \int_{-1}^{(3-r_U^2)/2r_U} \sqrt{1-x^2}\,\mathrm{d}x + 2\int_{(3-r_U^2)/2r_U}^{2-r_U} \frac{\sqrt{(r_U-x)^2}\sqrt{4-r^2-2r_Ux-x^2}}{2(r_Ux-1)} \left(r-\sqrt{\frac{(r_U^2+r_Ux-2)^2}{(r_U-x)^2}}\right)\mathrm{d}x. \quad (9)$$

Unfortunately, this integral is very messy. However, we can find this relative number of solutions for $r_U \in (1, 3)$ numerically and add in the simpler result for $r_U \in [0, 1]$ to obtain an unnormalized radial pdf. Finally, as before, we find the volume of the solid of revolution to normalize the curve so that it truly is a pdf:

$$V = \pi^2 + 2\pi\int_1^3 r_U S(r_U)\,\mathrm{d}r_U \approx 4\pi^2 \text{ to 7 decimal places.} \quad (10)$$

(The first term, π^2, is the volume from $r_U \in [0, 1]$.)

Therefore, in polar coordinates, the joint pdf for the first iteration of the aggressor game is

$$f(r_U, \theta) = \frac{S(r_U)}{\approx 4\pi^2} \quad (11)$$

which is plotted in Figure 9.

3.2 Expected Radial Distances

3.2.1 Defender Game To calculate the expected radial distances, we work in polar coordinates and choose random points A and B on a unit disk. Without loss of generality we can set $A \in (\sqrt{r_A}, 0)$ and $B \in (\sqrt{r_B}, \theta)$ where $\theta \in [0, 2\pi)$ (with θ measured clockwise from vertical) and $r_A, r_B \sim U(0, 1)$, and $\theta \sim (0, 2\pi)$. From these, the midpoint of AB is

$$M = \left(\frac{\sqrt{r_A + r_B - 2\sqrt{r_A r_B}\cos\theta}}{2}, \tan^{-1}\left(\frac{\sqrt{r_B}\sin\theta}{\sqrt{r_A} - \sqrt{r_B}\cos\theta}\right)\right). \quad (12)$$

Then,

$$\overline{r_M} = \frac{2\int_0^1\int_0^1\int_0^{2\pi}\sqrt{r_A + r_B - 2\sqrt{r_A r_B}\cos\theta}\;\mathrm{d}r_A\mathrm{d}r_B\mathrm{d}\theta}{2\int_0^1\int_0^1\int_0^{2\pi}\mathrm{d}r_A\mathrm{d}r_B\mathrm{d}\theta}. \quad (13)$$

Using the result from Uspensky (1937, pp. 257–258) and http://mathworld.wolfram.com/DiskLinePicking.html, we obtain

$$\overline{r_M} = \frac{198}{90\pi} = 0.452. \quad (14)$$

In contrast, the expected distance from the center of a set of points uniformly distributed across a disk is $\frac{1}{\sqrt{2}}(= 0.707)$. Therefore, the "pull" towards the center with the M's actual joint pdf is even stronger than with uniform across a disk.

3.2.2 Aggressor Game In the aggressor game, the polar coordinates of M are

$$M = \left(\sqrt{r_B + 4(r_A + \sqrt{r_A r_B}\cos\theta}, \tan^{-1}\left(\frac{\sqrt{r_B}\sin\theta}{2\sqrt{r_A} + \sqrt{r_B}\cos\theta}\right)\right). \quad (15)$$

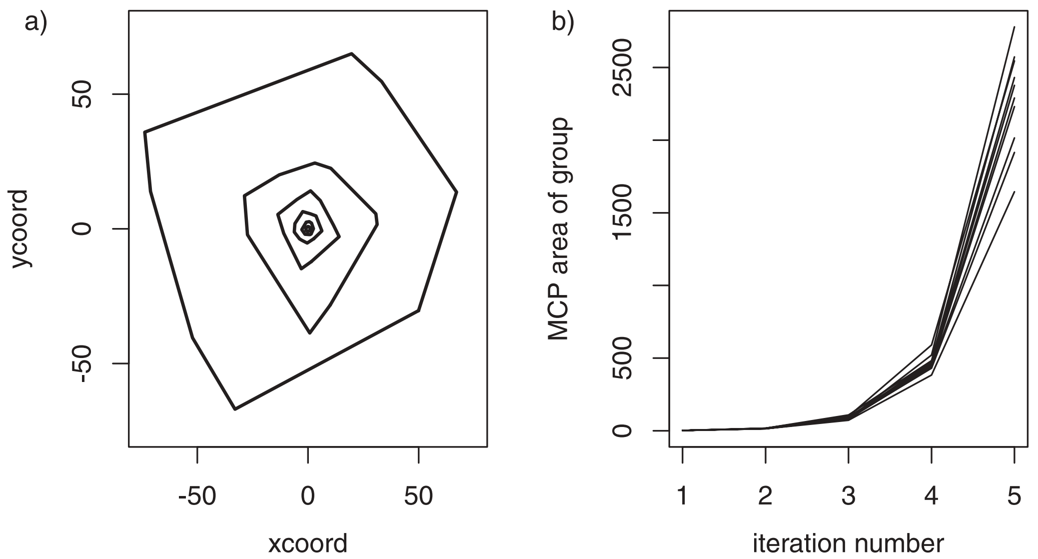

Figure 10 The rate of expansion for the aggressor game with n =100 participants: (a) minimum convex polygons of the participants' locations for five successive iterations of a single game, (b) MCP area versus iteration number (20 replicates).

Table 1 Expected radial distances for three distributions: The initial uniform and the first iteration of the defender and aggressor games. The value for the defender is less than that for uniform, and hence the group collapses, whereas the opposite is true for the aggressor game.

Distribution	Expected radial distance		
Uniform on a disk (= stalker game)	$1/\sqrt{2}$	=	0.707
Defender game	$128/90\pi$	=	0.452
Aggressor game	Equation 16	=	1.457

Then,

$$\overline{r_M} = \frac{\int_0^1\int_0^1\int_0^{2\pi}\sqrt{r_B + 4(r_A + \sqrt{r_A r_B}\cos\theta)}\,\mathrm{d}r_A\mathrm{d}r_B\mathrm{d}\theta}{\int_0^1\int_0^1\int_0^{2\pi}\mathrm{d}r_A\mathrm{d}r_B\mathrm{d}\theta}$$

$$= \frac{9.15466}{2\pi} = 1.457. \quad (16)$$

3.3 Rate of Expansion of the Group

As for the defender game (Section 2.3), I simulated the underlying process, in this case, however, to examine the rate of expansion. The results are shown in Figure 10. The expansion rate is so high that I only show five iterations. As for the defender game, the expansion is exponential, but here with a positive exponent: $0.535e^{1.683x}$ (R^2 = 0.997). The theoretical maximum exponent is 2ln3 = 2.197, a value larger than that observed, as expected.

3.4 Conclusions

In this game, the aggressor game maps an initial uniform on a disk (cylinder radius 1, height $1/\pi$), to essentially a frustum of a cone (radius 3, height of cone $4\pi/3$, height to cutoff of frustum $1/4\pi$). On average, an individual will end up farther from the center than from a uniform on a disk distribution, and may even end up as far as three units distance from *C*. Additional calculations provide the expected radial distance for the three scenarios (Table 1). As would be expected from our above calculations, not only are the expected radial distances ordered defender < uniform < aggressor, but the expected value for the aggressor game is especially high, thus implying rapid expansion.

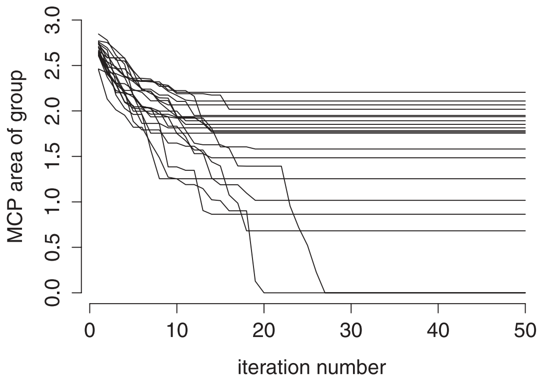

Figure 11 Stalker game. Each curve is a separate simulation of 100 participants and plots area of minimum convex polygon of group versus iteration number. Clearly, there is an initial collapse (complete collapse in 2 of the 20 replicates) and stability thereafter. In these simulations, individuals jump to *M* (see text).

4 Stalker Game

Earlier, I defined the new "stalker game" as one in which each participant *U* selected and pursued a single participant from the group. Despite being so simple—arguably the simplest game one could devise—these games (technically there are two forms: Chase *A* or chase *B*), are surprisingly complex. While it is true that $T(r_U) > 0.5$ for $r_U \in [0, 1]$—implying group collapse—it is also true that the joint pdf will be uniform on a disk—implying group stability. The joint pdf is easy to calculate, either by integration or by the same logic used in the other two games: For any point *M* on the disk, *any* point *A* (or *B*) on the disk satisfies the game rule, implying π "solutions" for $r_U \in [0, 1]$. Thus, the unscaled radial pdf is π and the joint pdf $f(r, \theta)$ is simply $1/\pi$ for $r \in [0, 1]$ and $\theta \in [0, 2\pi)$.

If there is a one-to-one mapping between the initial uniform distribution on a disk and the same uniform distribution at the end of the iteration, does the game really generate group collapse and if so, how? The answer is an interesting yes, it collapses, and yes, it is stable. Figure 11 plots MCP area versus iteration number for 20 replicates. There is distinct initial collapse for all replicates (completely so for two cases) but then stability thereafter. How can we explain these strange results?

There are two different answers depending upon an assumption or rather implementation detail of the game. I refer to the rate at which individuals reassess their heading. Real participants of the aggressor–defender games continuously adjust their heading to account for the movements of their *A* and *B* whereas in the simulation in Figure 11, each participant essentially jumps to their intended location *M* simultaneously.

First, let us suppose we have a finite group of participants; they each select their *A* (which is also their *M*), correct the heading, and a fraction of a second after they set off, we freeze frame the game. What has happened to the size of the group? Our earlier result $T(r_U) > 0.5$ tells us that individuals are more likely to head (roughly) towards the center of the disk than away. Therefore, our freeze frame image will show a group size *smaller* than the unit disk. Now we unfreeze the game and ask the participants to readjust their heading and set off again. No one will be heading outside this group, only within this reduced disk, and the distribution of headings across this reduced disk should be rotationally invariant (on average). Thus the same results holds true on this smaller disk: $T(r_U) > 0.5$. As such, when the reassessment rate is high, the group dispersion get smaller each iteration (but see below).

Now, we contemplate the other extreme: Instantaneous and simultaneous jumping from current location to intended location *M*. In this situation, we have a finite and fixed set of locations. Apart from the initial random locations and the random "partner" choices (*A*), this is a completely deterministic system. Thus, it may end up chaotic, in which case the MCP area of the group would be expected to fluctuate somewhat, or it may become periodic, or come to a fixed-point solution. In Figure 11 we observe the latter two outcomes (a chaotic system that produces exactly the same MCP area each iteration is highly unlikely). In the two cases that collapse completely, this is the fixed-point solution: All participants arrive at the same location (MCP area = 0). In the other 18 cases with stable MCP area, it seems that periodic behavior has arisen. This would imply that each participant moves around a circuit of locations. More detailed investigations (not shown) reveal that this is precisely what happens. (One problem with this scheme is that there is a high probability of fragmentation: That is, separate groups of participants chasing each other.) Why, though, should the system always collapse a little? A location at the perimeter would fail to be jumped onto if no-one had selected (= chasing) the participant at that location. For *n* participants, this probability is

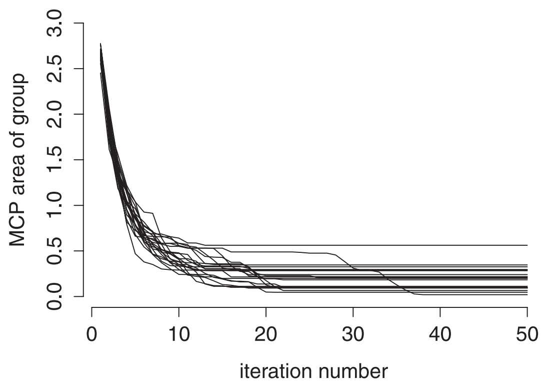

Figure 12 Stalker game with d = 0.25. 20 replicates with 100 participants.

$$\left(\frac{n-2}{n-1}\right)^{n-1}, \tag{17}$$

which quickly approaches its limit $1/e$, which is reasonably large. Thus, even for small *n*, almost certainly some of the outer participants fail to be selected; at the first jump they move inwards and the locations they leave behind are never jumped upon, and so the area of the group reduces.

Let $1/d$ represent the reassessment rate. Thus, an individual reassesses its heading after it has traveled length *d*, or it has reached *M*, whichever is the shorter (an implicit assumption is that all individuals are updated simultaneously). When the distance between a participant and *M* is greater than *d* then the group collapses (as described above). However, after a certain number of iterations, the size of the group will be sufficiently small that the distance to *M* is less than *d*, and so individuals, and finally the whole group, switch(es) to the jumping scenario above in which the size of the group stabilizes. This is clearly illustrated in Figure 12 where d = 0.25. For continuous reassessment, i.e., small *d*, the group is expected to collapse completely.

4.1 Conclusions

In this game, the aggressor game maps an initial uniform on a disk (cylinder radius 1, height $1/\pi$) to itself. What is key to the group dynamics is the interplay between the assessment rate and the result that $T(r_U) > 0.5$ (such that individuals tend to head towards or through the center): The combination of the uniform distribution on a disk and the individual readjustment essentially resets the game but with all the participants in a smaller and smaller disk. The extreme case is instantaneous jumping to the target. Here, the simple game rule that depends only a single target means that "jump point" locations are fixed. Approximately $1/e$ of the individuals will not be chosen, but will jump inwards, and so the boundary (MCP) must get smaller, at least initially, and thereafter settle to periodic behavior or complete collapse.

5 Discussion

My goal in this study was to provide some insight as to the link between the micro-level, that is, the individual level rules, and the macro-level, the group dynamics, of two simple spatial participative games. My geometrical approach, which yields some closed-form solutions, also leads to a new third game, the stalker game, which despite being incredibly simple also exhibits some rather interesting, and at first sight counterintuitive, behavior. In each case, we are able to provide some mapping between the initial distribution of participants and the joint pdf of their distribution after the first iteration. With such simple games, we can further argue (although I have not, unfortunately, yet provided formal proof) that similar mappings and feedback mechanisms will occur in subsequent iterations.

It seems that two pieces of information can hint strongly at what sort of behavior one might expect from the group. First, the probability of moving towards C: $T(r_U)$. Although both aggressor and defender games yielded probabilities greater than 0.5, this need not always be the case. For instance, a "repulsive" anti-stalker game whereby *U* moves to be say three times as far from *A* as you currently are, might yield a probability of moving towards *C* much less than 0.5. Second, the expected radial distance: this metric indicates how far from *C*, on average, participants end up after this first iteration and Table 1 shows significant differences among the three games. With an expected radial distance much greater than 1, as in the aggressor game, it is not surprising that such a rapid rate of expansion occurs. This analysis, therefore, might provide some hope of understanding the micro–macro level mappings in other, simple self-organized systems.

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